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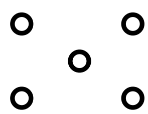
*Prompt:*

Students in an 8th grade prealgebra class had been investigating linear functions through patterns, developing recursive formulas, and writing direct formulas. One example follows:

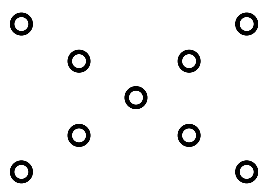
PATTERN #1

Consider the pattern generated by the following arrangement of dots:

At the beginning



At one minute



At two minutes

Students were asked the following questions to move them towards the construction of recursive and direct formulas:

i. Complete the following table giving the number of dots it takes to form the pattern at each minute.

| Minute # | # of dots |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

ii. If you knew how many dots were in the figure at a particular minute, how would you find out the number dots at the next minute?

iii. How many dots would it take to build the figure at the 50th minute?

iv. Find a formula for finding the number of dots it would take to build the figure if you knew only the minute number.

A couple of students responded to question (iii) by taking the number of dots at minute 5 and multiplying by 10 to get the number of dots at minute 50.

*Mathematical Foci 1: Proportional Reasoning*

It took the teacher a minute to figure out why the student would have thought to approach the problem in this way. In reflecting on this, it seems that the work the students had done to develop proportional reasoning (primarily through using ratio tables in a variety of settings to solve problems) led the student to treating this relationship as a proportional one. In a ratio table, students come to understand that you can maintain the relationship by multiplying (or dividing) both quantities by the same number or by adding two pairs of numbers (ratios) together. Understanding this, the question for the teacher became how to move the student from this understanding of a limited class of linear functions (proportions) to the entire set of linear functions.

*Mathematical Foci 2: Linear functions*

*Prompt:*

Students in an 8th grade prealgebra class encountered the following in their *Mathematics in Context* curriculum to develop their understanding of why division by zero is undefined while division into zero is zero.

The development began with a reminder of the two related division statements for a given multiplication statement. For example,

$3 \times 4 = 12$ and the related division statements $12 \div 3 = 4$ and $12 \div 4 = 3$

Students were then asked to consider the statements: $0 \times 4 = 0$ and $0 \times 5 = 0$ and write the related division statements:

$0 \div 4 = 0$ and $0 \div 0 = 4$ for $0 \times 4 = 0$

$0 \div 5 = 0$ and $0 \div 0 = 5$ for $0 \times 5 = 0$

The students were then asked, "How would you construct an argument for property that zero divided by any nonzero number is zero?" and "What is the problem with zero divided by zero?"

*Mathematical Foci 1: Properties of Real Numbers*
*Mathematical Foci 2: Division by zero; Division of zero*

The teacher is faced with several pressing questions—

- What instructional interventions would move students from specific examples to a more deductive proof of the property: *Zero divided by any nonzero number is zero.?*
- What would be the best way to deal with the given in this discussion: *zero times any number is zero*? Accept it and ignore it? Explain it? Justify it?
- What instructional interventions would move students from understanding $0 \div 0$ *is undefined* to $a \div 0$ *for nonzero a is undefined?*